Proper Solution of Circularity in the Interactions of Corporate Financing and Investment Decisions: A Reply to the Financing Present Value Approach

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Abstract

It is a well known problem the interactions between the market value of cash flows and the discount rate (usually the weighted average cost of capital, WACC) to calculate that value. This is mentioned in almost all textbooks in corporate finance. However, the solution adopted by most authors is to assume a constant leverage D%, and hence assume that the leverage gives raise to an optimal capital structure and the discount rate is constant.

On the other hand, most authors use the definition of the Ke, the cost of leveraged equity for perpetuities even if the planning horizon is finite. Among these authors we find the work of Wood and Leitch W&L 2004.

In this article we wish to analyse the claim made by W&L 2004 in the sense to have found an iterative solution to the problem of circularity that results in a "near" matching with the Adjusted Present Value APV, proposed by Myers, 1974. They use as the basic principle the fact that there is a "near" constant relation between Ke the cost of equity and Kd the cost of debt. They consider as well that the cost of debt Kd is not constant and changes proportionately with the leverage D%.

We propose a very simple and precise approach to solve the above mentioned circularity problem.

Keywords: Adjusted Present Value, APV, weighted average cost of capital, circularity problem, discounted cash flow, DCF equity value, cost of equity.

Introduction

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On the other hand, most authors use the definition of the Ke, the cost of leveraged equity for perpetuities even if the planning horizon is finite. Among these authors we find the work of Wood and Leitch W&L 2004.

Mohanti 2003 proposes an iterative method to solve the issue. Wood and Leitch 2004, (W&L) propose an iterative and approximate method to solve this circularity. Vélez-Pareja and Tham (2000), Tham and Vélez–Pareja, 2002, Vélez-Pareja and Burbano 2005 and Tham and Vélez–Pareja, 2004b have shown and proposed a very simple manner to tackle the issue of circularity.

Taggart 1991, Vélez-Pareja and Tham (2000), Tham, and Vélez–Pareja, 2002, Vélez-Pareja and Burbano 2005 and Tham, Vélez–Pareja, 2004a and 2004b, have derived independently the expression for Ke when there are finite cash flows.

In this paper we wish to analyse the claim made by W&L 2004 in the sense to have found an iterative solution to the problem of circularity that gives a "near" matching with the Adjusted Present Value, APV, proposed by Myers, 1974. They use as the basic principle the fact that there is a "near" constant relation between Ke the cost of equity and Kd the cost of debt, namely

$$K = \frac{1+Kd}{1+Ke} \tag{1}$$

They consider as well that the cost of debt Kd is not constant and changes proportionately with the leverage D%.

We propose a very simple and precise approach to solve the above mentioned circularity problem.

The paper is organised as follows: in Section One we present assumptions behind the WACC, the proper formulation for Ke, the cost of equity for finite horizons and the solution of the circularity with a simple example presented by W&L 2004, taken from Brealy and Myers (1982, p.337). In Section Two we use the same simple example to illustrate how to match the Net Present Value NPV, and the APV when using the proper definition of Ke and the proper value of a parameter proposed by W&L, 2004. Obviously, in the example we solve the circularity. In Section Three we conclude.

Section One

Cost of Capital, Assumptions for Finite Horizons and Perpetuities

Most finance textbooks (See Benninga and Sarig, 1997, Brealey, Myers and Marcus, 1996, Brealey and Myers 1982, 1996 and 2003, Copeland, Koller and Murrin, 1994, Damodaran, 1996, Gallagher and Andrew, 2000, Van

Horne, 1998, Weston and Copeland, 1992) present the Weighted Average Cost of Capital WACC calculation as:

$$WACC = Kd(1\text{-}T)D\% + KeE\% \quad (2)$$

Where Kd is the cost of debt before taxes, T is the tax rate, D% is the percentage of debt on total value, Ke is the cost of equity and E% is the percentage of equity on total value. All of them precise that the values to calculate D% y E% are market values. Although they devote special space and thought to calculate Kd and Ke, little effort is made to the correct calculation of market values. In fact equation (2) is the most common expression for the weighted average cost of capital, WACC. This means that there are several points that are not sufficiently dealt with:

1. Market values are calculated period by period and they are the present value at WACC of the future cash flows.
2. These values used to calculate D% and E% are located at the beginning of period t, where the WACC belongs.
3. Kd(1-T) implies that the tax payments coincides in time with tax accruals. (Some firms could present this payment behavior, but it is not the rule. Only those that are subject to tax withheld from their customers, pay taxes as soon as they accrue income taxes).
4. Because of 1., 2., and the existence of changing macroeconomic environment, (say, inflation rates) WACC changes from period to period.
5. That there exists circularity when calculating WACC. In order to know the firm value it is necessary to know the WACC, but to calculate WACC, the firm value and the financing profile are needed.
6. That we obtain full advantage of the tax savings in the same year as taxes are paid. This means that earnings before interest and taxes (EBIT) are greater than or equal to the interest charges.
7. That the only source of tax savings is the interest charges.
8. That (2) implies a definition for Ke usually for finite periods, the cost of equity, in this case,

$$Ke_i = Ku_i + (Ku_i - Kd_i)\left(\frac{D_{i-1}}{E^L_{i-1}} - \frac{V^{TS}_{i-1}}{E^L_{i-1}}\right) \quad (3a)$$

In this expression, Ke_i is the leveraged cost of equity, Ku_i is the cost of unleveraged equity, Kd is the cost of debt, V^{TS}_{i-1} is the value of the tax savings discounted at the proper rate of discount (in this case that rate is Kd) at the

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previous period, D_{i-1} is the value of debt at i-1 and E_{i-1} is the market value of equity at i-1 and expression (3a) is valid for perpetuities or finite cash flows. It is a general formulation of Ke. When the cash flow is a perpetuity we obtain (assuming Kd as the discount rate for the tax savings) the traditional expression for Ke, namely

$$Ke_i = Ku_i + (Ku_i - Kd_i)(1 - T)\left(\frac{D_{i-1}}{E^L_{i-1}}\right) \quad (3b)$$

Where T is the corporate tax rate and other variables as defined above.

Our approach to the solution of the circularity that arises from the interactions of corporate financing and investment decisions is very simple: it is based on a recursive equation and on the feature of iteration that spreadsheets have. The recursive equation is:

$$V_i = \frac{CF_{i+1} + V_{i+1}}{1 + WACC_{i+1}} \quad (4)$$

Where CF is the cash flow at i+1, V is the present value at the given period, and WACC is the weighted average cost of capital at period i+1. Care has to be taken at the start of the iterative process not to divide by zero when calculating WACC. This is done *starting with a WACC set as a number* (zero or any other value). Done this, we activate the iteration feature of the spreadsheet and formulate the WACC as indicated in (1). The activation of the feature is described based on a Microsoft Excel spreadsheet, as follows:

1. Select the option *Tools* in the textual menu in Excel.
2. Select *Options*
3. Select the tab *Calculate*.
4. In the dialog box select *Iteration* and click *Ok*.

This procedure can be done before starting the work in the spreadsheet or when Excel declares the presence of circularity. After these instructions are done, then, the WACC can be calculated as the sum of the debt and equity contribution to the cost of capital. It is recommended that the last arithmetic operation be the WACC calculation as the sum of the debt and equity contribution to the cost of capital.

In the case of perpetuities, in order to calculate the PV of the FCF using the WACC we have to use the proper expression for it. When the discount rate for tax savings is Kd, the proper expression for WACC for non-growing perpetuities is (see Taggart 1991, Tham and Vélez-Pareja, 2002 and 2004b and Vélez-Pareja and Burbano, 2005):

$$WACC_i - Ku - \frac{KuTD_{i-1}}{V_{i-1}} \qquad (5)^2$$

and

$$V_{i-1} = \frac{FCF_i}{WACC_i} = \frac{FCF_i}{Ku - \dfrac{KuTD_{i-1}}{TV_{i-1}}} \qquad (6)$$

Where D is the value of debt and other variables have been defined above.

We have a problem here and it is that we need V. However, from (6) we can solve for V and then

$$V_{i-1}\left(Ku - \frac{KuTD_{i-1}}{V_{i-1}}\right) = FCF_i \qquad (7a)$$

Developing left hand side of equation, we have

$$V_{i-1}Ku - KuTD_{i-1} = FCF_i \qquad (7b)$$

Grouping terms,

$$V_{i-1}Ku = FCF_i + KuTD_{i-1} \qquad (7c)$$

And finally solving for V,

$$V_{i-1} = \frac{FCF_i + KuTD_{i-1}}{Ku} = \frac{FCF_i}{Ku} + TD_{i-1} \qquad (7d)$$

But this is the definition of the Adjusted Present Value, APV!

An Example for Calculating WACC and the Firm Value

For a better understanding of these ideas, an example is presented. We use the two periods example proposed by W&L 2004, taken from Brealy and Myers, 1982. Assume a firm with the following information: cost of the unleveraged equity Ku, 13%, Kd is 10% and tax rate 40%. Initial investment is 100 and initial debt is 50. Full principal payment is done in the end of year 2. Discount rate for tax savings is Kd.[3]

If debt is not traded, the cost is the one stipulated in the contract.

The information about the investment, free cash flows, debt balances, initial equity, principal payment interest, tax savings and its present value at Kd are

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Table 1 Free cash flow and initial investment			
Year	**0**	**1**	**2**
Free cash flow FCF		74.00	74.00
Debt at end of period, D	50.00	50.00	0.00
Initial equity investment	50.00		
Total initial investment	100.00		
Principal payments		0.00	50.00
Interest charges		5.00	5.00
Tax savings TS		2.00	2.00
V^{TS} at Kd	3.47	1.82	

The WACC calculations are done estimating the debt and equity participation in the total value of the firm for each period and calculating the contribution of each to the WACC after taxes. As a first step, we will not add up these components to find the value of WACC and we will calculate the total firm value with the WACC set at 0. We will construct each table, step by step, assuming that WACC is zero. Remember that $D_{t-1}\% = D/V_{t-1}$.

Table 2 WACC calculation. Contribution of debt to WACC			
Year	**0**	**1**	**2**
Debt			
Debt at end of period, D	50.00	50.00	0.00
Relative weight of debt D% (Debt balance)/Total value of firm at t-1) (Calculated using V from table 4, below)	33.78%	67.57%	
Cost of debt Kd		10.00%	10.00%
Cost of debt after taxes Kd(1-T)		6.00%	6.00%
Contribution of debt to WACC Kd(1-T)D%		2.03%	4.05%

The same procedure is used to estimate the contribution of equity to WACC.

For the calculation of Ke we have to remind that Total value is debt plus equity. Hence, Equity is Total value minus debt.

Table 3 WACC calculation. Contribution of equity to WACC			
Year	**0**	**1**	**2**
Equity			
Relative weight of equity E% = 1- D%	66.22%	32.43%	
V^{TS} at Kd	3.47	1.82	
Cost of equity Ke = Ku + (Ku-Kd)((D-V^{TS})/(V-D))		14.42%	21.06%
Contribution of equity to WACC = E%× Ke		9.55%	6.17%

Done this, our table for WACC and Total Value will appear as

Table 4 WACC calculations			
Year	**0**	**1**	**2**
FCF		74.00	74.00
WACC after taxes (Debt + equity contributions)			
Total value V, at t-1 and WACC = 0	148.00	74.00	

In our example, firm value at end of year 1 is

$$VT_1 = \frac{FCF_2 + VT_2}{1 + WACC_2} = \frac{74.00 + 0.00}{1 + 0\%} = 74.00$$

For year 0 it will be

$$VT_0 = \frac{FCF_1 + VT_1}{1 + WACC_1} = \frac{74.00 + 74.00}{1 + 0\%} = 148.00$$

Now we can proceed to formulate the WACC as the sum of the two components: debt contribution and equity contribution. When the WACC is calculated, table 4 is shown as table 5

Table 5 WACC calculation. Contribution of debt to WACC (final)			
Year	**0**	**1**	**2**
Debt			
Debt at end of period, D	50.00	50.00	0.00
Relative weight of debt D% (Debt balance)/(V of firm at t-1)	39.40%	74.29%	
Cost of debt Kd		10.00%	10.00%
Cost of debt after taxes Kd(1-T)		6.00%	6.00%
Contribution of debt to WACC Kd(1-T)D%		2.36%	4.46%

The same procedure is used to estimate the contribution of equity to WACC.

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Table 6 WACC calculation. Contribution of equity to WACC (final).			
Year	**0**	**1**	**2**
Equity			
Relative weight of equity E% = (1-D%)	60.60%	25.71%	
Cost of equity Ke = Ku + (Ku-Kd)((D-V^{TS})/(V-D))		14.81%	21.35%
Contribution of equity to WACC = E%× Ke		8.98%	5.49%

Note that the cost of equity –Ke– is larger than Ku as expected, because Ku is the cost of the stockholder as if the firm were unleveraged. When there is debt necessarily Ke ends up being greater than Ku, because of leverage. With these values it is possible to calculate the firm value for each period.

Table 7 WACC calculations (final)			
Year	**0**	**1**	**2**
FCF		74.00	74.00
WACC after taxes (Debt + equity contributions)		11.34%	9.95%
Firm value a end of t	126.91	67.30	

Notice that WACC results in a lower value than Ku. WACC is after taxes. In our example, firm value at end of year 1 is

$$VT_1 = \frac{FCF_2 + VT_2}{1 + WACC_2} = \frac{74.00 + 0.00}{1 + 9.95\%} = 67.30$$

For year 0 it will be

$$VT_{i0} = \frac{FCF_1 + VT_1}{1 + WACC_1} = \frac{74.00 + 67.30}{1 + 11.34\%} = 126.90$$

The reader has to realize that the values 11.34% and 9.95% are not calculated from the beginning because they depend on the firm value that is going to be calculated with the WACC. In this case circularity is generated. This is solved allowing the spreadsheet to make enough iterations until it finds the final numbers.

With the WACC values for each period the present value of future cash flows and the NPV are calculated.

Table 8 NPV calculations	
Year	**0**
Present value of cash flows	126.91
NPV	26.91

If the initial investment is 100, then, NPV is 26.91.

Using the Myers approach on the case with taxes the same result can be reached calculating the present value for the free cash flow and discounting it at Ku, and the present value of the tax savings calculated at Kd, the cost of debt. Myers proposed this in 1974 and it is known as Adjusted Present Value APV.

Table 9 Calculation of value and APV	
Year	**0**
PV(FCF at Ku)	123.44
PV(TS at Kd)	3.47
PV(FCF at Ku) + PV(TS at Kd) = APV	126.91
Adjusted NPV	26.91

As can be seen, the total values are identical. Notice that the same result is reached with the two methods. All these calculations should coincide. In this example,

Table 10 A comparison of values by different approaches		
Method	**Total Value**	**Equity Value = Total Value - Debt**
PV(FCF at $WACC_t$)	126.91	76.91
PV(FCF at Ku) + PV(Tax savings at Kd)	126.91	76.91

Notice that all the values match.

In this section we have shown the procedure to calculate the present value of a cash flow solving the circularity problem. This procedure is fully consistent with APV. This means that we have solved the interactions between corporate financing and investment decisions (circularity).

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Section Two

Simple Example to Show the Solution

In this Section we use the example presented by W&L2004, to illustrate how solving the circularity with the proposed algorithm and using the proper Ke formulation we can match the APV and the NPV using the FCF and the WACC.

In this section we show the example proposed by W&L 2004 and show four cases:

1. Kd variable and R_f(risk free rate) constant n= 2 (Reference case, presented by W&L2004)
2. Kd variable and R_f(risk free rate) variable n=2
3. Kd variable and R_f(risk free rate) variable n=3.156756714
4. Kd variable and R_f(risk free rate) constant n= 2.263741895

The parameters for the example are mentioned in Section One. For Ke we use (3a), above. For Ke, W&L use (3b), above. We use the equations proposed by W&L for R_f and Kd as follows

$$R_f = \frac{Kd - KdD\%^n}{1 - D\%^n} = \frac{Kpar - KuD\%^n}{1 - D\%^n} \quad (8)$$

and

$$Kd = R_f + (Ku - R_f)D\%^n \quad (9)$$

Where Kd is the debt supplier's required rate of return, Kpar is the coupon interest rate on par value of debt, D% is the leverage at market value, R_f is the risk free rate and Ku is the unleveraged cost of equity.

In Case 1 we fix R_f constant and let Kd to vary as a function of the D% for each year. If we fix R_f and Kd then R_f is 9% and Kd is the original value of Kd, namely 10% and the values for DCF and APV are identical as shown in Section One.

Adjusted NPV, ANPV, is 27.04. Differences between levered value and APV are 0.00693043 for year 0 and 0.02152035 for year 1. In their article, W&L report a difference at year 0 of 0.75 and it is attributed to the fact that "the two methods are discounting similar but not identical cash flows".

Table 11 Case 1. Kd variable and R_f (risk free rate) constant n= 2 (Reference case, presented by W&L2004)			
Year	**0**	**1**	**2**
FCF		74.00	74.00
Debt at end of period, D	50.00	50.00	0.00
Interest		4.81	5.60
TS		1.92	2.24
V^{TS}	3.60	2.04	
R_f		9.00%	9.00%
Kd		9.62%	11.19%
Kd(1-T)		5.77%	6.72%
D%		39.40%	74.10%
Kd(1-T)D%		2.27%	4.98%
E%		60.60%	25.90%
Ke		15.04%	17.95%
KeE%		9.12%	4.65%
WACC		11.39%	9.63%
Levered value	127.03	67.50	
NPV	27.03		
PV(FCF at Ku)	123.44	65.49	
APV = PV(FCF at Ku) + V^{TS}	127.04	67.52	

Next we develop Case 2, the same example assuming that R_f and Kd are not constant, this is that D% is variable and Kd and R_f depend each year from the D% calculated at market values. We assume n = 2 as before.

Table 12 Case 2. Kd variable and R_f (risk free rate) variable n=2			
Year	0	1	2
FCF		74.00	74.00
Debt at end of period, D	50.00	50.00	0.00
Interest		4.77	6.30
TS		1.91	2.52
V^{TS}	3.82	2.29	
R_f		9.45%	6.41%
Kd		9.55%	12.59%
Kd(1-T)		5.73%	7.56%

D%		39.30%	73.83%
Kd(1-T)D%		2.25%	5.58%
E%		60.70%	26.17%
Ke		15.06%	14.09%
KeE%		9.14%	3.69%
WACC		11.40%	9.27%
Levered value	127.23	67.72	
NPV	27.23		
PV(FCF at Ku)	123.44	65.49	
APV = PV(FCF at Ku) + V^{TS}	127.26	67.78	

Adjusted NPV, ANPV is 27.26. This time differences are 0.03126 for year 0 and 0.05258 for year 1. Theses differences are well below the differences reported by W&L.

Table 13 Case 3 Kd variable and R_f (risk free rate) variable, n= 3.156756714

Year	0	1	2
FCF		74.00	74.00
Debt at end of period, D	50.00	50.00	0.00
Interest		4.59	5.78
TS		1.84	2.31
V^{TS}	3.58	2.10	
R_f		9.45%	6.37%
Kd		9.19%	11.56%
Kd(1-T)		5.51%	6.94%
D%		39.36%	74.01%
Kd(1-T)D%		2.17%	5.14%
E%		60.64%	25.99%
Ke		15.30%	16.91%
KeE%		9.28%	4.40%
WACC	11.45%	9.53%	
Levered value	127.02	67.56	
NPV	27.02		
PV(FCF at Ku)	123.44	65.49	
APV = PV(FCF at Ku) + V^{TS}	127.02	67.59	

Now we develop Case 3. In this case we have defined Kd and R_f as a function of the D% for each year. We define n in a way to match APV and DCF values.

Adjusted NPV, ANPV is 27.02. In this case we modified n, setting the differences to zero (at year 0) and using Solver having as changing variable n. Now the differences are 0.00000000 for year 0 and 0.02911787 for year 1. Again, differences are well below the one reported by W&L for year 0.

For case 4 we define R_f as a function of a constant D% (50%) and Kd as a function of D% for each year, as presented by W&L 2004. We define n in a way that APV and DCF match.

Table 14 Case 4. Variable Kd and constant R_f (risk free rate)
n= 2.263741895

Year	**0**	**1**	**2**
FCF		74.00	74.00
Debt at end of period, D	50.00	50.00	0.00
Interest		4.74	5.51
TS		1.90	2.21
V^{TS}	3.55	2.01	
R_f		9.00%	9.00%
Kd		9.48%	11.03%
Kd(1-T)		5.69%	6.62%
D%		39.37%	74.10%
Kd(1-T)D%		2.24%	4.90%
E%		60.63%	25.90%
Ke		15.12%	18.41%
KeE%		9.17%	4.77%
WACC		11.41%	9.67%
Levered value	126.99	67.47	
NPV	26.99		
PV(FCF at Ku)	123.44	65.49	
APV = PV(FCF at Ku) + V^{TS}	126.99	67.49	

Adjusted NPV, ANPV is 26.99. As in Case 3, we modified n, setting the differences to zero (at year 0) and using Solver having as changing variable n. Now the differences are 0.00000000 for year 0 and 0.018271 for year 1. Again, differences are well below the one reported by W&L for year 0.

Regarding cases 3 and 4, it is questionable that a method has to define the value of a parameter as 2.263741895 or 3.156756714. More questionable is that we have to arrive to those values using tools like Solver; this situation allows that some mistakes are made in the modeling process, but result disguised when we force the result to match (matching the discounted cash flow DCF result with the APV result) values and find the value of a parameter. Why should n be 2, 2.263741895 or 3.156756714? There is no analytic or intuitive answer to this question. In the first case (2) we do not obtain consistent results as is shown in the W&L paper and with the results shown in the previous tables; regarding the last two values there is no human being that could imagine beforehand the value of those parameters in order to match the results. In the case of our proposed algorithm however, the results match if the proper inputs are given, and it has to be said that when we say match we mean that the results should be identical. As was shown when the four cases were presented, the fact that Kd depends on the value (becoming a new source of circularity) makes it necessary to properly define the value of the input parameters (including n), otherwise the matching (identical values) will never arise, even with the proposed procedure.

As a summary, we present the next table:

Table 15 Summary of calculations

	Kd constant	Case 1*	Case 2**	Case 3***	Case 4****
Levered value DCF	126.91	127.03	127.23	127.02	126.99
Levered value APV	126.91	127.04	127.26	127.02	126.99
n	N/A	2.00	2	3.156756714	2.263741895
Difference in value at yr 0	-	(0.0069304)	(0.0312557)	0.0000000	0.0000000

* Case 1 = Kd variable and R_f constant n= 2 (Reference case, presented by W&L2004)
** Case 2 = Kd variable and R_f (risk free rate) variable n=2
*** Case 3 = Kd variable and R_f (risk free rate) variable n=3.156756714
**** Case 4 = Kd variable and R_f constant n= 2.263741895

In all cases the differences are well below the one reported by W&L 2004. It has to be said that the major source of differences is the formulation for Ke. When Ke is formulated for perpetuities as in (3b) and is used for finite cash flows APV and DCF will *never* match.

In the Benninga and Sarig (1997, p.260-262) example, T is 22%, Kd is 5%, Ku is 12.5%, and FCF is $10,000 forever. Debt is 40,000. The APV is calculated as 88,000 as usual:

$$PV(FCF\ at\ Ku) = \frac{FCF}{Ku} = \frac{10{,}000}{12.5\%} = 80{,}000^4$$

$$TS = 40{,}000 \times 5\% \times 22\% = 440$$

$$PV(TS\ at\ Kd) = \frac{TS}{Kd} = \frac{440}{5\%} = 8{,}800$$

$$APV = 88{,}800$$

Using (7d), V for perpetuities is

$$TV_{i-1} = \frac{10{,}000 + 12.5\% \times 22\% \times 40{,}000}{12.5\%} = \frac{10{,}000 + 1{,}100}{12.5\%} = \frac{11{,}100}{12.5\%} = 88{,}800$$

Or if we prefer, assuming an arbitrary value for WACC, in this case, 12%, we can solve this inducing a circularity as

$$TV_{i-1} = \frac{10{,}000}{WACC_{perp}} = \frac{11{,}000}{12.5\%} = 83{,}333.3333$$

When we introduce the correct formula in the spreadsheet, we obtain

$$TV_{i-1} = \frac{10{,}000}{Ku - \frac{KuTD}{TV_{i-1}}} = \frac{11{,}000}{11.26126126\%} = 88{,}800.00$$

Again, APV for perpetuities and DCF with WACC match, they are identical and there is no circularity.

In the Finnerty (1986, p.150-158) example cited by W&L 2004 we have the following parameters: D% constant of 1/3 (this means that in the model D is set as 1/3 of the levered value for the firm), R_f constant defined by (5), Kd defined by (6), Ku is 15%, Kd is 12%, T is 46% and n is 2. With these inputs we constructed the following table where Ke is defined as in (3a) and the circularity is solved using our recursive equation (4) with the above mentioned procedure.

APV and DCF are identical for the 10 years. This result coincides in a payment (PMT) of 5.6 for year 10 as in the Finnerty example. When the definition of Ke is as in (3b) we find exactly the PMT of 5.62 reported by W&L and the levered value does not match with APV as expected, because Ke is defined for perpetuities. It is not "due to rounding or cutoff errors" in R_f, Kd or D% as W&L claim. As a side comment, in this case Kd (and V) is invariable for n.

Table 16. Finnerty example presented by W&L 2004

Year	0	1	2	3	4	5	6	7	8	9	10
FCF		19	19	19	19	19	19	19	19	19	19
D=D%×V	34.39	32.51	30.39	27.99	25.29	22.25	18.81	14.93	10.55	5.60	-
PMT		1.89	2.12	2.39	2.70	3.04	3.44	3.88	4.38	4.95	5.60
Interest		4.13	3.90	3.65	3.36	3.04	2.67	2.26	1.79	1.27	0.67
TS		1.90	1.79	1.68	1.55	1.40	1.23	1.04	0.82	0.58	0.31
R_f		11.63%	11.63%	11.63%	11.63%	11.63%	11.63%	11.63%	11.63%	11.63%	11.63%
Kd		12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%	12.00%
Kd(1-T)		6.48%	6.48%	6.48%	6.48%	6.48%	6.48%	6.48%	6.48%	6.48%	6.48%
D%		1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3	1/3
KdD%(1-T)		2.16%	2.16%	2.16%	2.16%	2.16%	2.16%	2.16%	2.16%	2.16%	2.16%
Ke		16.16%	16.18%	16.21%	16.24%	16.26%	16.29%	16.32%	16.36%	16.39%	16.43%
E%		2/3	2/3	2/3	2/3	2/3	2/3	2/3	2/3	2/3	2/3
KeE%		10.77%	10.79%	10.81%	10.82%	10.84%	10.86%	10.88%	10.90%	10.93%	10.95%
WACC		12.93%	12.95%	12.97%	12.98%	13.00%	13.02%	13.04%	13.06%	13.09%	13.11%
V	103.18	97.53	91.16	83.98	75.88	66.75	56.44	44.80	31.65	16.80	
E=V-D	68.79	65.02	60.77	55.98	50.59	44.50	37.63	29.87	21.10	11.20	
V^{TS}	7.83	6.87	5.90	4.93	3.98	3.06	2.20	1.42	0.77	0.28	
PV(FCF at Ku)	95.36	90.66	85.26	79.05	71.91	63.69	54.24	43.38	30.89	16.52	
V= APV	103.18	97.53	91.16	83.98	75.88	66.75	56.44	44.80	31.65	16.80	

It has to be said that when properly done, *all* methods for calculating firm or equity value *have* to give the same results. More, the sum of the present value (PV) of free cash flow (FCF) at Ku and the PV of the tax shield (TS) at Kd must equal the sum of the PV of the cash flow to debt (CFD) and the PV of the cash flow to equity (CFE). See Vélez-Pareja and Tham, 2000, 2003a, 2003b, 2004a, 2004b and 2004c, Tham and Vélez-Pareja 2004c, Vélez-Pareja and Burbano, 2003.

Section Three

Concluding Remarks

We have shown a very simple procedure to solve the circularity induced by the interactions of corporate financing and investment decisions. This is a real practical a simple solution that analyst might easily implement. Circularity is not solved in the sense that we live with it in the model, but it is solved in the sense that given the circularity we perform, using an appropriate spreadsheet, the iterations that allow us to reach the consistent values of all the variables that define or cause the circularity.

Using the proper definition of Ke for finite cash flows, the proper definition of WACC for perpetuities, assuming that the discount rate for the tax savings is Kd and applying the proposed procedure to handle the circularity, we show that the differences reported by W&L 2004 are relatively large compared with the differences found with our method which are negligible or strictly equal to zero. We can say that the major differences found by W&L are due to the use of (3b) as a definition of Ke which is appropriate for perpetuities but is applied to finite cash flows, and a wrong definition of WACC for perpetuities. On the other hand, W&L compare their findings with an APV calculated with constant Kd. APV has to be calculated with the same Kd used to calculate the interest payments and tax savings, and hence if Kd is not constant, that has to be reflected in the calculation of the present value of the tax savings.

Finally, it has to be said that when properly done, *all* methods for calculating firm or equity value *have* to give identical results.

Endnotes

1. It has to be said that this concept is one of the most mentioned and used concept in corporate finance, but we do not know someone that has in reality seen that ethereal and elusive concept.

2. It can be shown that the expressions for WACC at perpetuity for non growing perpetuities and when the discount rate for the TS is Kd and for the present value of a cash flow in perpetuity are $Ku_r - \frac{Ku_r \times T \times Kd \times D\%}{Kd - g}$ and $\frac{FCF_N}{WACC_r}$ respectively; however, we will keep the definition (5) that is consistent with the example by Benninga and Sarig, below and proposed by W&L. See Vélez-Pareja and Tham 2005 and Appendix A for recalculation of the value in the example.

3. In this example we assume Kd constant. However, as it is shown in Appendix B, Kd might be variable.

4. See Appendix for the calculation with a different definition of WACC for perpetuity.

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APPENDIX A

Proper Solution of Circularity in the Interactions of Corporate Financing and Investment Decisions

(This Appendix is based on Vélez-Pareja, Ignacio and Joseph Tham, 2005, WACC, Value of Tax Savings and Terminal Value for Growing and Non Growing Perpetuities, Working Paper en SSRN, Social Science Research Network, September. (August), http://papers.ssrn.com/abstract =789025)

In this appendix we will derive the expressions for the present value of a cash flow in perpetuity, the expression for the WACC in perpetuity and the expression for the V^{TS} in perpetuity. All these derivations are made under the assumption of Kd as the discount rate for the TS.

The Present Value of a Non Growing Perpetuity

First of all we have to define the value of a non growing perpetuity in terms of the real rate of growth.

We assume that the real growth rate $g_r > 0$ and the FCFs are in perpetuity, the debt is risk-free, that is $Kd = R_f$ and the leverage D% is constant. As we said above, we are concerned with real growth, growth due to inflation does not require new investment, and hence the general expression for the levered value of a growing perpetuity is

$$V^L = \frac{NOPLAT_{N+1}\left(1-\frac{g_r}{ROIC_r}\right)}{WACC-g} = \frac{NOPLAT_N(1+g)\left(1-\frac{g_r}{ROIC_r}\right)}{WACC-g} \quad (A1)$$

Where V^L is the present value of the firm in perpetuity, g is the nominal growth rate, g_r is the real growth rate, $ROIC_r$ is the real return on invested capital, WACC is the nominal weighted average cost of capital in perpetuity and $NOPLAT_N$ is the net operating profit less adjusted taxes at year N as an estimation of FCF that allows the assets to be constant (when NOPLAT is used as FCF it is assumed that the depreciation in invested at perpetuity)..

. The $g_r/ROIC_r$ is the fraction of the FCF that has to be invested in order to keep the FCF growing at g_r.

We have to make an statement regarding ROIC at perpetuity. If we assume ROIC to be greater than WACC at perpetuity it might be too optimistic. If we assume it is smaller than WACC perpetuity it might be too pessimistic. A reasonable assumption is that ROIC equals WACC in perpetuity (this is a stable state condition). If we assume that $ROIC_r = WACC_r$ then

$$V^L = \frac{NOPLAT_N(1+g)\left(\frac{WACC_r-g_r}{WACC_r}\right)}{WACC-g} \quad (A2)$$

As we have assumed and explained that FCF = NOPLAT and why we do this, we will use the FCF notation understanding that when we use FCF it means NOPLAT.

When there is inflation there might be to drivers to keep FCF growing: one is the real growth g_r, that makes the FCF in real terms (say in units of product or service) and the inflation that makes the FCF grow in nominal terms. This growth is offset by the discounting process. Hence, the only growth we are concerned with in terms of the amount to be invested in order to keep the FCF growing at perpetuity is g_r.

When we realize that it is a non growing perpetuity we should either consider that there is no real growth and no inflation or there is real growth and inflation. To assume that there is no inflation is not realistic, however, with or without inflation, the expression for the present value of a perpetuity is the same (See Vélez-Pareja and Tham, 2005).

$$V^l = \frac{FCF_{N+1}}{WACC - i} = \frac{TCF_N(1+i)}{WACC - i} \qquad \text{(A3a)}$$

Expressing WACC in terms of $WACC_r$ and inflation rate i we have

$$V^L = \frac{FCF_N(1+i)}{(1+WACC_r)(1+i)-1-i} = \frac{FCF_N(1+i)}{1+WACC_r + i + i \times WACC_r - 1 - i} \qquad \text{(A3)}$$

Simplifying we have

$$V^L = \frac{FCF_N(1+i)}{WACC_r + i \times WACC_r} = \frac{FCF_N(1+i)}{WACC_r(1+i)} = \frac{FCF_N}{WACC_r} \qquad \text{(A3c)}$$

We see that the V^L at N with no growth is not $V^L = \frac{FCF_N}{WACC}$ as usual, discounting the perpetuity with the nominal WACC. The same derivation can be used to calculate the unlevered value, V^L.

The Real WACC in Perpetuity

In the same line, we have to derive the $WACC_r$ at perpetuity.

From above we know that

$$V^L = \frac{FCF_N}{WACC_r} \qquad \text{(A4a)}$$

Solving for FCF_N

$$FCF_N = V^L \times WACC_r \qquad \text{(A4b)}$$

On the other hand for the case of the unlevered firm we assume $ROIC_r = Ku_r$

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$$V^{Un} = \frac{FCF_N}{Ku_r} \qquad \text{(A5a)}$$

Solving for FCF_N we have,

$$FCF_N = V^{Un} Ku_r \qquad \text{(A5b)}$$

Equating equations A4b and A5b, we obtain,

$$V^L WACC_r = V^{Un} Ku_r \qquad \text{(A5c)}$$

As

$$V^{un} = V^L - V^{TS} \qquad \text{(A6)}$$

then

$$V^L WACC_r = (V^L - V^{TS}) \times Ku_r \qquad \text{(A7a)}$$

Dividing by V^L and reorganizing terms we have

$$WACC_r = Ku_r \left(1 - \frac{V^{TS}}{V^L}\right) \qquad \text{(A7b)}$$

We know that

$$V^{TS} = \frac{TS}{\psi - g} \qquad \text{(A8)}$$

Where g is the growth rate for TS (and CFD) and ψ is the discount rate for the TS.

Then

$$WACC_r = Ku_r \left(1 - \frac{TS}{(\psi - g)V^L}\right) \qquad \text{(A9)}$$

This is the general formulation. For the case of non growing perpetuities, inflation and ψ = Kd, we have

$$WACC_r = Ku_r \left(1 - \frac{Kd \times T \times D}{(Kd - i_f)V^L}\right) \qquad \text{(A10a)}$$

$$WACC_r = Ku_r \left(1 - \frac{Kd \times T \times D\%}{(Kd - i_f)}\right) \qquad \text{(A10b)}$$

$$WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{((1+Kd_r)(1+i)-1-i)} \qquad (A10c)$$

$$WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{(1+Kd_r + (i + i \times Kd_r)-1-i)} \qquad (A10d)$$

$$WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{(Kd_r + i \times Kd_r)} \qquad (A10c)$$

$$WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{Kd_r(1+i)} \qquad (A10f)$$

The VTS for Perpetuities

From the previous derivations we can now see what the V^{TS} is for perpetuities:

We know that

$$V^{TS} = V^{L} - V^{Un} \qquad (A11)$$

$$V^{TS} = \frac{FCF_N}{WACC_r} - \frac{FCF_N}{Ku_r} \qquad (A12a)$$

$$V^{TS} = FCF_N \frac{Ku_r - WACC_r}{WACC_r \times Ku_r} \qquad (A12b)$$

$$V^{TS} = V^{L} \frac{Ku_r - WACC_r}{Ku_r} \qquad (A12c)$$

V^{TS} for this case

($g_r = 0$ $i > 0$ $\psi = Kd$ and $WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{Kd_r(1+i)}$) can be calculated as

$$V^{TS} = V^{L} \frac{Ku_r - Ku_r + \frac{Ku_r \times Kd \times T \times D\%}{Kd_r(1+i)}}{Ku_r} \qquad (A13a)$$

$$V^{TS} = \frac{Ku_r \times K_d \times T \times D}{Kd_r(1+i) \times Ku_r} \qquad (A13b)$$

$$V^{TS} = \frac{Ku_r \times T \times D}{Kd_r(1+i)} \qquad (A13c)$$

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In the Benninga and Sarig (1997, p.260-262) example, T is 22%, Kd is 5%, Ku is 12.5%, and FCF is $10,000 forever. Debt is 40,000. We have to include an extra assumption: inflation rate. Let us assume that inflation rate, i, is 4%. In that case we have to estimate Ku_r and Kd_r.

Table A1. Calculation of APV for $g_r = 0$ i> 0 ψ = Kd	
Kd	5.00%
Ku	12.50%
i	3.00%
Kd_r	1.94%
Ku_r	9.22%
FCF	10000
T	22%
$\frac{FCF}{Ku_r}$	108,421.05
$V^{TS} = \frac{Kd \times T \times D}{Kd_r(1+i)}$	22,000.00
$APV = \frac{FCF}{Ku_r} + \frac{Kd \times T \times D}{Kd_r(1+i)}$	130,421.05

Using a circular relationship and solving the circularity, we have,

Table A2. Calculation of value with circularity	
D	40,000.00
D% = D/V	30.67%
$WACC_r = Ku_r - \frac{Ku_r \times Kd \times T \times D\%}{(Kd_r(1+i)}$	7.67%
$V = FCF/WACC_r$	130,421.05

As can be seen, the two values match as expected.

Appendix B

In this Appendix we use the same example as in Table 1 in the body of the paper. The difference is that Kd varies as in the W&L paper:

$$Kd = R_f + (Ku - R_f)*D\%^2 \quad \text{(B1)}$$

Where R_f is the risk free rate, Ku is the unlevered cost of equity and D% is the corresponding leverage. We assume ψ = Kd. Initial investment, I, is 100.

Table B1. Input for example

FCF	74	Kd	10%
TS	2	Ku	13%
D	50	R_f	9%
T	40%	I	100

Now we calculate the value and the NPV using the solution of circularity discounting the FCF with the WACC and using the APV.

Table B2. Value using discounting FCF with WACC

	Year 0	Year 1	Year 2
FCF		74.00	74.00
TS		1.92	2.24
Interest		4.81	5.60
D	50.00	50.00	
Principal payment		-	50.00
CFD		4.81	55.60
CFE = FCF + TS - CFD		71.11	20.64
V^{TS}	3.59	2.01	
R_f		9.00%	9.00%
$Kd = R_f + (Ku-R_f) \times D\%^2$		9.62%	11.19%
Kd(1-T)		5.77%	6.72%
D%		39.36%	74.07%
Kd(1-T)D%		2.27%	4.98%
E%		60.64%	25.93%
Ke		15.04%	17.95%
KeE%		9.12%	4.65%
WACC = Kd(1-T)D% + KeE%		11.39%	9.63%
Levered value	127.03	67.50	
NPV	27.03		

Now we calculate the value with the APV.

Table B3. Calculating value with the APV

	Year 0	Year 1
PV(FCF at Ku)	123.44	65.49
V^{TS}	3.59	2.01
APV	127.03	67.50
ANPV	27.03	

As expected, the results match.